

## COEFFICIENT OF CONSOLIDATION FOR SOIL – THAT ELUSIVE QUANTITY

DONG WANG<sup>\*</sup>, MARK F. RANDOLPH<sup>\*</sup> AND SUSAN GOURVENEC<sup>\*</sup>

<sup>\*</sup> Centre for Offshore Foundation Systems  
The University of Western Australia  
35 Stirling Highway, Perth, WA 6009, Australia

e-mails: [dong.wang@uwa.edu.au](mailto:dong.wang@uwa.edu.au), [mark.randolph@uwa.edu.au](mailto:mark.randolph@uwa.edu.au), [susan.gourvenec@uwa.edu.au](mailto:susan.gourvenec@uwa.edu.au)  
[www.cofs.uwa.edu.au](http://www.cofs.uwa.edu.au)

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**Abstract.** Although it is accepted that the coefficient of consolidation for soil is not a true material property, but reflects the net effect of permeability and compressibility, it is a very useful parameter in day to day design. Design calculations make extensive use of elastic solutions for consolidation, such as beneath a shallow foundation or around a driven pile, but an important consideration is how to measure or estimate an appropriate coefficient of consolidation to use in those solutions. Typically the quantity is determined either from laboratory oedometer tests (generally then referred to as  $c_v$ ) or from field dissipation tests using a piezocone or piezoball penetrometer (generally then referred to as  $c_h$ ). Since the latter form of test includes a mix of stress paths, for some of which the soil has a stiffness associated with unloading and others of which involve plastic compression, the magnitude of  $c_h$  for a given soil is typically 3 to 10 times the value of  $c_v$  from virgin compression in laboratory oedometer tests. The paper explores the relationship between  $c_v$  and  $c_h$  for different boundary value problems, within the confines of soil modelled as Modified Cam Clay, for both isotropic and anisotropic permeability. Problems range among: simulated oedometer testing, field dissipation testing and pore pressure response beneath a shallow foundation. Results of finite element analysis of this range of problems are used to develop guidelines for different classes of problem, comparing the relevant coefficient of consolidation against a benchmark  $c_v$  value associated with virgin compression in an oedometer. The normalised values of consolidation coefficient are expressed as functions of fundamental soil parameters used within Modified Cam Clay.

### 1 INTRODUCTION

The coefficient of consolidation,  $c_v$ , was originally presented in Terzaghi's classical one-dimensional consolidation theory to estimate foundation settlement under vertical loading. The coefficient is usually determined through laboratory oedometer or Rowe cell tests, or from in situ pore pressure dissipation tests with piezocone or piezoball. In each type of test, the value of consolidation coefficient is deduced by comparison of measured data with a theoretical response curve, generally derived from simple elastic response of the soil. In an oedometer or Rowe cell test, the one-dimensional compression of the soil sample for a given stress increment

is plotted against either the square root of time or the logarithm of time in order to deduce  $c_v$  [1]; in a field dissipation test, the excess pore pressure decay is plotted against the logarithm of time and matched to a theoretical response [2] in order to deduce an ‘operative’ coefficient of consolidation, usually expressed as  $c_h$ . No formal relationship between  $c_h$  and  $c_v$  has been established, although in practice the former is generally found to be significantly greater, by a factor of 3 to 10, than the latter. In both laboratory and field tests, uncertainties arise because of the need to normalise the measured responses, identifying appropriate initial and final values of the measured parameters.

In this paper, the coefficient of consolidation is interpreted from either finite strain or large deformation finite element (FE) analyses that incorporate an elastoplastic critical state soil model, Modified Cam Clay (MCC). The objective is to identify relevant ‘operative’ values of consolidation coefficient for different boundary value problems, devising relationships between the different values in terms of true soil parameters. Comparisons are made with experimental data obtained from laboratory and centrifuge model tests at the University of Western Australia, using normally consolidated kaolin clay, for which MCC parameters are well established. All FE simulations have been conducted in the framework of finite strain rather than small strain, using the commercial package Abaqus/Standard [3]. The soil is discretised with 8-node axisymmetric elements with reduced integration and pore pressures at 4 corner nodes (termed CAX8RP in Abaqus).

## 2 SOIL PROPERTIES

The properties of the kaolin clay considered in this study are listed in Table 1. The soil sample is assumed to have isotropic permeability ( $k_h/k_v = 1$ , where subscripts  $h$  and  $v$  indicate horizontal and vertical directions), as has been demonstrated in various centrifuge tests [4,5]. The effect of anisotropic permeability on dissipation of excess pore pressure will also be explored numerically.

According to results from Rowe cell tests [6], the coefficient of consolidation for normally consolidated conditions may be fitted as:

$$c_v = \sqrt{0.001 + 0.014\sigma'_v / p_a} \text{ mm}^2/\text{s} \quad (1)$$

where  $\sigma'_v$  is the vertical effective stress and  $p_a$  is atmospheric pressure (100 kPa). The coefficient of consolidation was determined through the square root method (Taylor’s method). For normally consolidated soil under a  $K_0$  state, the relationship between void ratio and vertical effective stress within the MCC model is

$$e = (e_N - C) - \lambda \ln \left( \frac{1 + 2K_0}{3} \sigma'_v \right) \quad (2)$$

where  $C$  is the distance between the virgin consolidation line (VCL) and one-dimensional normal consolidation line, and  $K_0$  is the coefficient of earth pressure at rest. Equations 1 and 2 allow the permeability of the kaolin to be determined as a function of the void ratio  $e$ :

$$k_v = \gamma_w m_v c_v = 1.06 \times 10^{-5} \frac{\gamma_w \lambda (1 + 2K_0)}{(1 + e) \exp((e_N - C - e)/\lambda)} \sqrt{1 + \frac{0.42 \exp((e_N - C - e)/\lambda)}{1 + 2K_0}} \text{ mm/s} \quad (3)$$

where  $\gamma_w$  is the unit weight of water and  $m_v$  is the coefficient of volume compressibility. The relationship for permeability given by Eq. 3 has been used successfully in recent numerical studies [5,7]. However, the accuracy of this equation is limited for two reasons: (i) the void ratio in a one-dimensional test varies with stress level and time during a consolidation stress increment; (ii) the coefficient of consolidation is determined from an empirical method (in this case Taylor's root time method rather than the log time method of Casagrande).

**Table 1:** MCC parameters for kaolin clay

Properties (after [8])	Values
Angle of internal friction, $\phi'$	23°
Void ratio at $p' = 1$ kPa on virgin consolidation line, $e_N$	2.252
Slope of normal consolidation line, $\lambda$	0.205
Slope of swelling line, $\kappa$	0.044
Plastic compression ratio, $\Lambda = 1 - \kappa/\lambda$	0.79
Poisson's ratio, $\nu$	0.3
Submerged unit weight, $\gamma'$	6.18 kN/m <sup>3</sup>

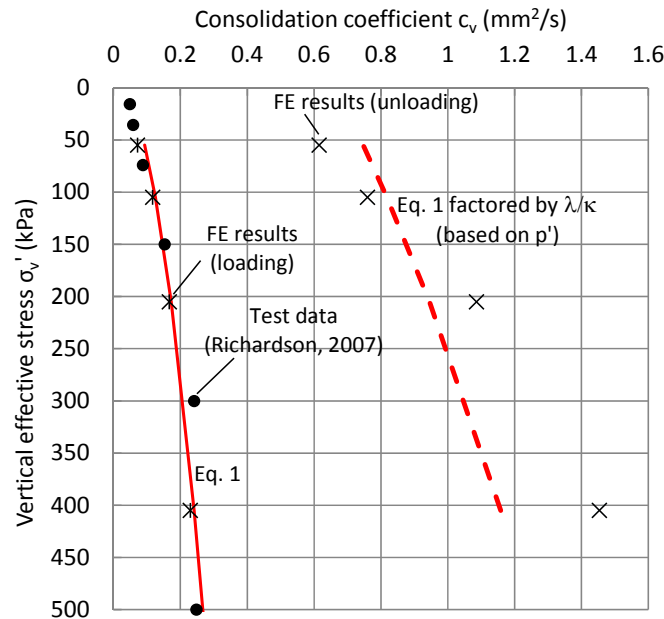
### 3 OEDOMETER TEST

To verify the effectiveness of Eq. 3, a conventional oedometer test was simulated using finite strain FE analysis. The soil sample was 75 mm in diameter and 20 mm high, with permeable top and bottom faces. The container was assumed fully smooth. Prior to one-dimensional consolidation, a small pressure of 5 kPa was applied on the soil surface to generate initial effective stresses within soil sample, with  $K_0$  selected as 0.75 rather than the more usual estimate of  $1 - \sin\phi' (= 0.61)$ . The reason is that the shape of the MCC yield envelope, assuming associated flow, automatically leads to a ratio between radial and vertical effective stress during one-dimensional compression between 0.7 ~ 0.8. The initial void ratio was thus calculated as  $e_0 = 1.942$  and  $C$  in Eq. 2 was 0.017. The vertical stress was increased from 5 kPa to 55, 105, 205, 405 and 505 kPa. Each stage lasted 8 h, which was sufficiently long for full dissipation of excess pore pressures. The elastic part of the MCC model was described with constant Poisson's ratio of 0.3. Once the simulation was completed, the value of  $c_v$  corresponding to each loading stage was derived through the square root time method.

Figure 1 shows the variations of  $c_v$  from Eq. 1 and from the FE simulation. Agreement is reasonable, suggesting that the permeability can be quantified through Eq. 3 with satisfactory accuracy. Terzaghi's 1-D consolidation theory is based on the simplifications that the soil is homogeneous and the permeability is constant during each loading stage. Additionally, there is always some divergence of  $c_v$  obtained through different empirical methods. The good agreement highlighted in Figure 1 is probably due to the reason that both the coefficient of compressibility

$$m_v = \frac{\lambda}{(1+e)\sigma'_v} \quad (4)$$

and permeability depend on the current void ratio, but the void ratio is varied in a relatively small range for each loading stage.

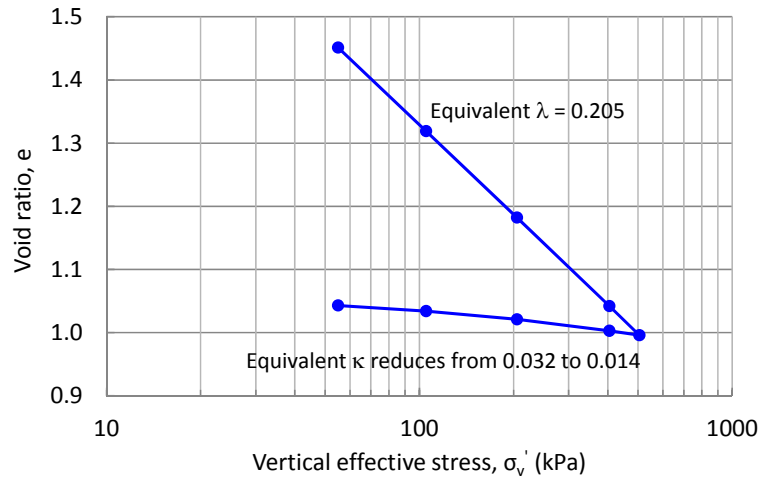


**Figure 1:** Coefficient of consolidation estimated by FE

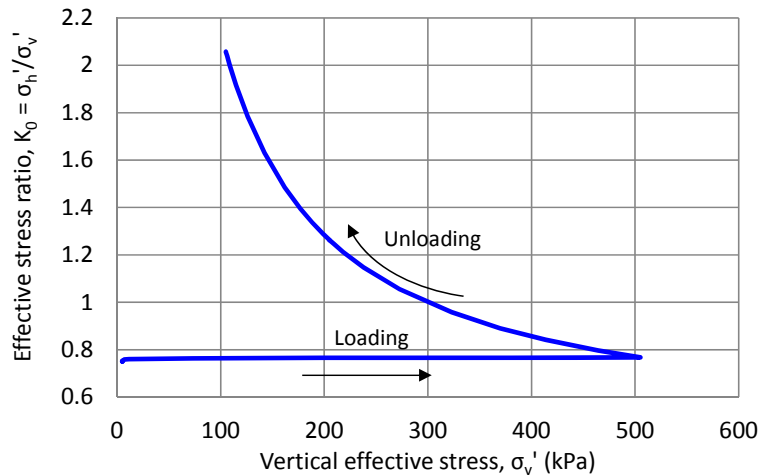
The unloading phase was also simulated, reducing the vertical stress applied to the soil surface in stages to 405, 205, 105 and 55 kPa. The resulting  $c_{v,unloading}$  values are shown in Figure 2, where they are compared with Eq. 1, factored by  $\lambda/\kappa$ . (Note the equation was recast to be based on  $p'$ , taking account of the changing  $K_0$  during 1-D unloading.) As may be seen, the FE data are broadly similar to the factored equation, but with a greater variation with respect to effective stress level. In fact it is difficult to ‘predict’ the operative consolidation coefficient for each stage of unloading, due to complex changes in the effective stress level,  $K_0$  and void ratio that take place (non-uniformly with time through the sample), as illustrated below.

The compression and swelling curves from the loading and unloading stage are shown in  $e - \sigma'_v$  space in Figure 2. While the loading response matches exactly the input value of  $\lambda$ , the unloading response shows a gradient that gives equivalent ‘ $\kappa$ ’ values (but with respect to  $\sigma'_v$  rather than  $p'$ ) that decrease from 0.032 to 0.014, compared with the input value of  $\kappa = 0.044$ . The discrepancy is due to the gradually increasing  $K_0$  value during 1-D unloading, as shown in Figure 3, and indeed the  $e - p'$  response matches exactly  $\kappa = 0.044$ . The difference in deduced values of  $\kappa$  obtained in either  $e - \sigma'_v$  or  $e - p'$  space is an aspect that is generally over-looked in deriving input parameters for soil models.

Although the above discussion is somewhat pedestrian in that it covers well-known aspects of soil response, it is included in order to provide background data that illustrate the rather complex nature of the consolidation coefficient, as applied to even very simple problems. The following sections extend the study to explore ‘operative’ values of consolidation coefficient obtained from two different applications, consolidation around a penetrometer, and consolidation beneath a shallow foundation.



**Figure 2:** Compression and swelling curves from FE simulation of oedometer test



**Figure 3:** Effective stress ratio during numerical reproduction of compression and swelling

#### 4 PIEZOBALL DISSIPATION TESTS IN NORMALLY CONSOLIDATED CLAY

Full-flow penetrometers such as the T-bar [9] and ball [10] have been used increasingly during the last decade, in particular in soft seabed sediments due to their advantages of larger projected area and minimal correction for overburden pressure. A ball penetrometer fitted with pore pressure transducer, a so-called piezoball [10], is of special interest as it can provide information on the soil consolidation properties by means of dissipation tests. Compared with one-dimensional consolidation laboratory test, the piezoball dissipation test is carried out in natural soil, without the need to obtain (nominally) undisturbed soil samples for later testing in the laboratory. This is especially important for offshore practice, since the cost of obtaining high-quality samples for laboratory testing are very high, and the low shear strengths in the upper few metres of the seabed pose particular practical difficulties.

In this section, the dissipation of excess pore pressure adjacent to the piezoball, following penetration, is studied using a large deformation finite element (LDFE) approach. The aim is to

establish a procedure for estimating  $c_h$  from the dissipation response for general anisotropic soil permeability. The LDFE approach is based on frequent mesh regeneration, in order to overcome soil element distortion induced by penetration of the ball. The details of the LDFE approach can be found in [11,12].

A centrifuge test [13] was reproduced to verify the reliability of the LDFE approach, assuming isotropic permeability, followed by investigation of the effect of anisotropic permeability. The model piezoball had a diameter of  $D_b = 15$  mm and shaft diameter of  $d = 5$  mm; the polypropylene filter element was fitted at either the mid-face (half the radius vertically from the tip of the ball probe) or the equator position; and the tests were conducted in normally consolidated kaolin at an acceleration level of 110g. The piezoball was penetrated to depth of 160 mm (estimated  $\sigma'_v \sim 109$  kPa) at a rate of 1 mm/s, and then the dissipation test was conducted maintaining position of the piezoball. To simplify the analyses, avoiding major distortion of the soil surface, the piezoball in the LDFE analyses was pre-embedded at a depth of 130 mm before penetrating by 2 diameters, rather than simulating penetration from the soil surface. The shafted-piezoball was simplified as fully smooth. The coefficient of earth pressure was taken as  $K_0 = 1 - \sin\phi' = 0.61$ , and  $C$  in Eq. 2 was calculated as 0.048.

To explore the effect of anisotropic permeability, the ratio between horizontal and vertical permeability,  $n = k_h/k_v$ , was changed from 1 to 2, 5 and 10. Note that in the LDFE simulations,  $k_h$  and  $k_v$  were in terms of global Cartesian coordinates. The penetration rate used in the test, 1 mm/s, was sufficient to ensure essentially undrained conditions for  $n = 1$ . However, our trial calculations showed that the penetration phase may allow some partial consolidation for  $n > 1$ . The penetration rate was thus increased to 10 mm/s in the LDFE simulations. The pore pressures induced in soil with  $n = 1$  are not affected significantly by the increase of penetrate rate, since the responses under undrained conditions are similar. Only the dissipations at mid-face of the ball are discussed here, as this position was deemed superior to the equator position for estimating  $c_h$  [7].

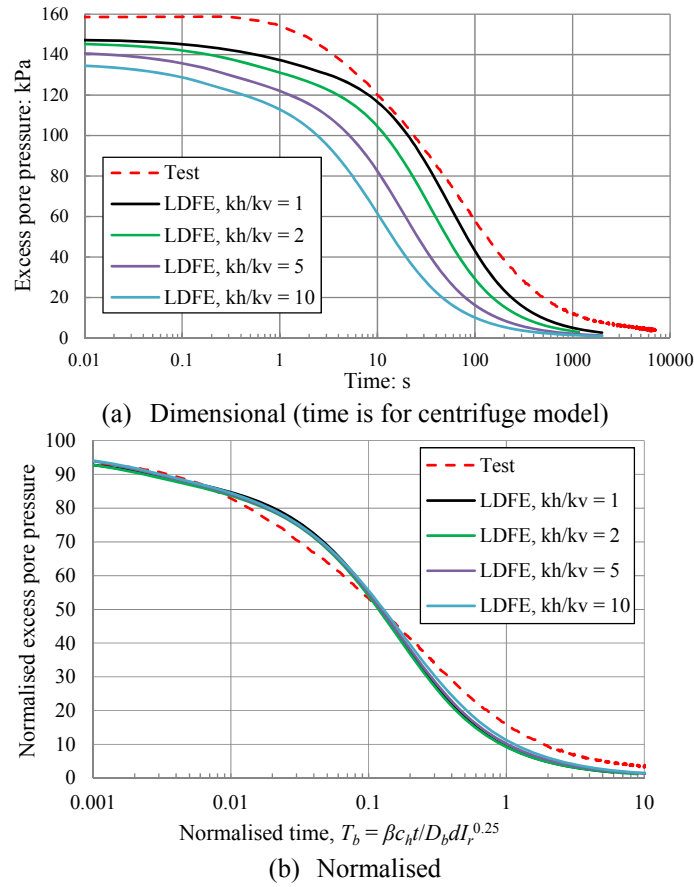
All the numerical dissipation responses are shown with the experimental data in Figure 4a. The numerical excess pore pressure at the end of penetration reduces from 147 to 135 kPa as  $n$  is increased from 1 to 10. This slight divergence suggests that nearly undrained conditions are achieved for permeability ratios no larger than 10. For soil with isotropic permeability, the measured excess pore pressure at the start of dissipation is slightly higher than the FE result. [7] suggested that the excess pore pressure is best normalised with an idealised initial excess pore pressure  $\Delta u_{ext}$  which is estimated using a back-extrapolation technique based on the square root of time [14]. An extensive parametric study, assuming isotropic permeability led to the suggestion that the consolidation time is best normalised as

$$T_b = \frac{c_h t}{D_b d I_r^{0.25}} \quad \text{with} \quad (5)$$

$$c_h = \frac{3(1-\nu)}{(1+\nu)} \left( \frac{\lambda}{\kappa} \right)^\alpha c_v = \frac{3(1-\nu)}{(1+\nu)} \frac{(1+e)p'}{\kappa^\alpha \lambda^{1-\alpha}} \frac{k_v}{\gamma_w} \quad (6)$$

where  $c_h$  is the operative coefficient of consolidation,  $I_r$  the rigidity index (equal to 73 here),  $p'$  the initial mean effective stress at the dissipation depth and  $\alpha$  is a fitting parameter selected as 0.75. Note that this value of  $\alpha$ , closer to unity than zero, reflects the stress paths followed during

the dissipation process, which tend to include a high component of (elastic) ‘reloading’, even for the initially normally consolidated soil conditions [7].



**Figure 4:** Dimensional and normalised dissipation responses at mid-face of piezoball

The normalised time from Eq. 5 is only for isotropic permeability. For permeability ratio  $n > 1$ , Eq. 5 may be modified as

$$T_b = \frac{\beta c_h t}{D_b d I_r^{0.25}} \quad (7)$$

with the factor  $\beta$  fitted as

$$\beta = \frac{n}{2} + \frac{1}{2} \quad (8)$$

The resulting normalised dissipation graphs are shown in Figure 4b. By introducing the factor  $\beta$ , the normalised numerical graphs with  $n$  ranging through 1 ~ 10 become nearly unique, establishing the effectiveness of the normalisation. Equation 7 indicates that the rate of dissipation at the mid-face of the piezoball depends on the average permeability in the vertical and horizontal directions. In contrast, the dissipation at the shoulder ( $u_2$  position) of the piezocone is generally assumed to be governed by the horizontal permeability.

The unique normalised dissipation graph provided in Figure 4b can be used to estimate  $c_v$

once the permeability ratio in a particular field is known. In situ test data for the dissipation of excess pore pressure as a function of time are interpreted through the following procedure:

(1) A nominal initial excess pore pressure,  $\Delta u_{ext}$ , at the start of dissipation is evaluated using the root time back-extrapolation technique [14].

(2) The dissipation time is then normalised using Eq. 7 (assisted by Eq. 8) with an assumed operative coefficient of consolidation  $c_h$ . The value of  $c_h$  is adjusted until the experimental normalised dissipation graph matches the numerical curves, such as those in Figure 4b. Note that it is necessary to assume a permeability ratio  $n$  a priori, or to undertake appropriate laboratory tests to evaluate the degree of permeability anisotropy.

(3) The coefficient of consolidation  $c_v$  at the dissipation depth of piezoball is estimated as (see Eq. 6):

$$c_v = \frac{(1+\nu)}{3(1-\nu)} \left( \frac{\kappa}{\lambda} \right)^\alpha c_h \quad (9)$$

A final comment relates to the application of the coefficient of consolidation to assess the degree of drainage during moving boundary problems, such as a penetration test [13,15]. The normalised velocity

$$V = \frac{vD}{c_v} \quad (10)$$

where  $v$  is the actual velocity and  $D$  is the relevant dimensions (e.g. diameter of penetrometer) may be used to quantify the degree of consolidation occurring during the motion. Generally, limits of  $V < 0.3$  for fully drained conditions and  $V > 30$  for undrained conditions are applied. However, it seems logical that these limits should really be expressed in terms of the same coefficient of consolidation as determined in a pore pressure dissipation test. In the case of a piezoball, that would change the limits for partial consolidation to (about)  $0.1 < V' < 10$ , where  $V' = vD/c_h$ . These limits match the experimental data of [13].

## 5 CHOICE OF CONSOLIDATION COEFFICIENT IN FOUNDATION DESIGN

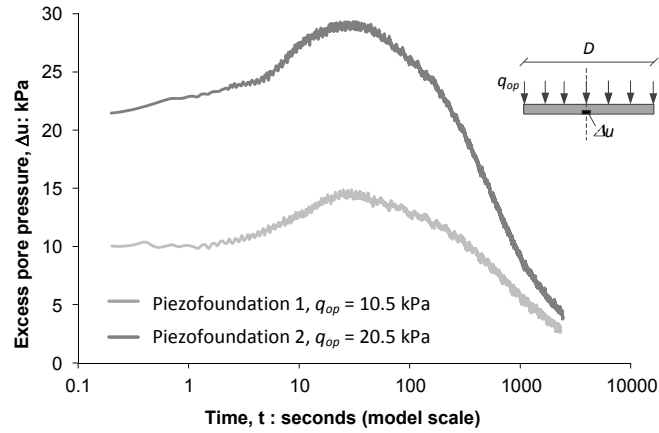
Just as the operative coefficient of consolidation  $c_h$  for a piezocone or piezoball dissipation test is greater than the corresponding  $c_v$  value from a laboratory consolidation test, so the relevant coefficient for excess pore pressure dissipation or consolidation settlement beneath a shallow foundation may differ from  $c_v$  because of the range of stress paths followed in the underlying soil.

The operative coefficient of consolidation governing dissipation beneath a rigid, circular foundation resting on the surface of normally consolidated kaolin clay was investigated through centrifuge model tests [16]. The so-called ‘piezofoundation’ comprised a rough, rigid circular plate equipped with a pore pressure sensor at the centre of the baseplate. Two dissipation tests were carried out at constant (average) vertical stress levels of 10.5 kPa (PF1) and 20.5 kPa (PF2). The dissipation time histories from each test are shown in Figure 5a.

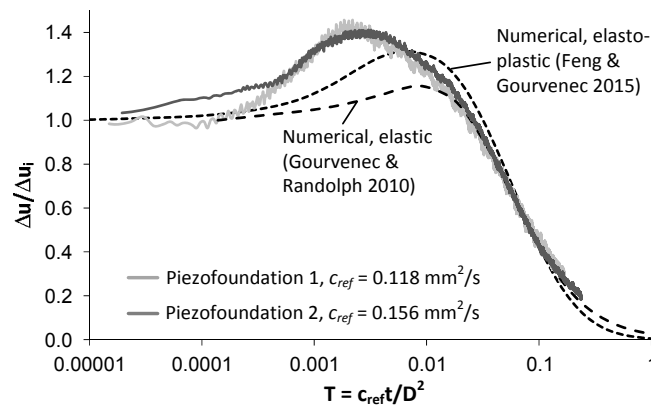
The piezofoundation tests exhibit the characteristic Mandel-Cryer effect with excess pore pressure increasing above the initial value of stress change during the early stage of consolidation. The Mandel-Cryer effect is a stress transfer effect significant in three-dimensional consolidation, resulting from the more rapid dissipation of excess pore pressure in



the soil near the edges of the foundation than near the centre. The resulting (greater) compression of the soil at the edges leads to a temporary (initial) transfer of total stress and hence increase in excess pore pressure in the central part of the foundation.



(a) Measured excess pore pressure dissipation



(b) Measured and numerical prediction of normalised excess pore pressure dissipation

**Figure 5:** Dissipation time histories for piezofoundation tests

Values of the operative coefficient of consolidation were determined by fitting the normalised measured dissipation curves to solutions based on elastic and elasto-plastic (Modified Cam Clay soil with isotropic permeability) finite element analyses of a rough, rigid, surface plate [17,18], as illustrated in Figure 5b. The excess pore pressure was normalised by the initial value and the consolidation time by a dimensionless time factor  $T = c_{ref}t/D^2$ , where  $D$  is the diameter of the foundation ( $= 40$  mm). Operative values of the coefficient of consolidation,  $c_{ref}$ , of  $0.12 \text{ mm}^2/\text{s}$  and  $0.16 \text{ mm}^2/\text{s}$  for tests PF1 and PF2, with foundation loads of  $10.5 \text{ kPa}$  and  $20.5 \text{ kPa}$  respectively, were deduced in order to fit the numerically derived solutions.

Complementary piezocone tests were carried out in the centrifuge sample in order to relate the operative coefficient of consolidation for the foundation response,  $c_{ref}$ , to that measured by the piezocone,  $c_h$ . The piezocone data was normalised with the idealised initial excess pore

pressure value,  $\Delta u_{ext}$ , estimated using back-extrapolation based on the square root of time method [14], as applied to the piezoball results presented in Section 4. The consolidation time was normalised as a time factor, denoted  $T^*$  [2], similar to that applied to the piezoball interpretation presented in Section 4.

$$T^* = \frac{c_h t}{R^2 I_r^{0.5}} \quad (11)$$

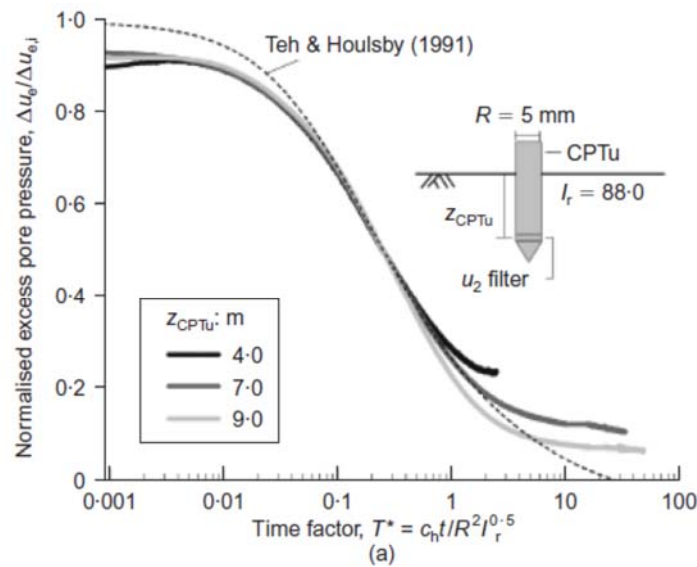
where  $R$  is the piezocone radius and  $I_r$  is a rigidity index, taken as 88, following [19]. The normalised time histories are shown in Figure 6. Values of the coefficient of horizontal coefficient of consolidation,  $c_h$ , were extrapolated from these dissipation curves using a correlation with an established theoretical solution based on  $T^*_{50}$ , the time for 50% excess pore pressure dissipation [2]. Values range from  $0.24 < c_h \text{ (mm}^2/\text{s)} < 0.41$  and are plotted as a function of vertical effective stress corresponding to the depth of each piezocone test in Figure 7, along with the operative values of the coefficient of consolidation,  $c_{ref}$ , from the foundation tests.

Additional piezocone results from other testing programmes in normally consolidated kaolin at UWA but carried out at higher stress levels [4, 20] are also shown on Figure 7, along with the prediction of  $c_v$  from  $c_h$  using Eq. 1. The ratio  $c_h/c_v$  is about 4.5, in good agreement with observations [4, 13]. The stress dependent relationships of  $c_{ref}$  and  $c_h$  can be approximated by power laws of the form given for  $c_v$  in Eq. 1. The relationships shown in Figure 7 indicate the operative coefficient of consolidation,  $c_{ref}$ , governing the response of a surface circular foundation under vertical loading, can be taken as  $0.6c_h$ , as measured from in situ piezocone (or piezoball) tests or  $2.6c_v$ , as measured through laboratory oedometer tests.

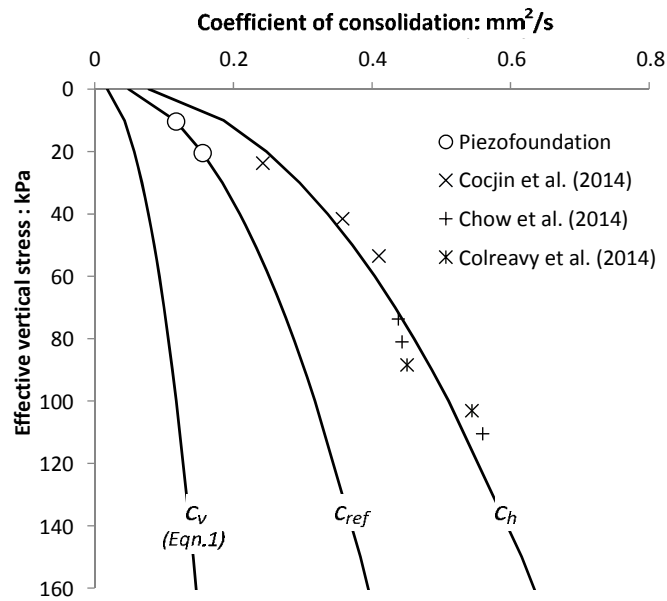
There is a curious feature of this comparison between the two numerical solutions and the experimental data. On the one hand the two numerical solutions coincide if the consolidation coefficient,  $c_v$ , from the MCC analysis is based on the virgin (plastic) 1-D compression modulus,  $M$ , so

$$c_v = \frac{kM}{\gamma_w} = \frac{k}{\gamma_w} \frac{(1+e)\sigma'_{v0}}{\lambda} \quad (12)$$

On the other hand, the deduced  $c_{ref}$  from the experimental data lies closer to the dissipation consolidation coefficient,  $c_h$ , than the value,  $c_v$ , associated with plastic 1-D compression. This apparent inconsistency needs further investigation, but again emphasises the complexity of assessing suitable value of consolidation coefficient for different boundary value problems.

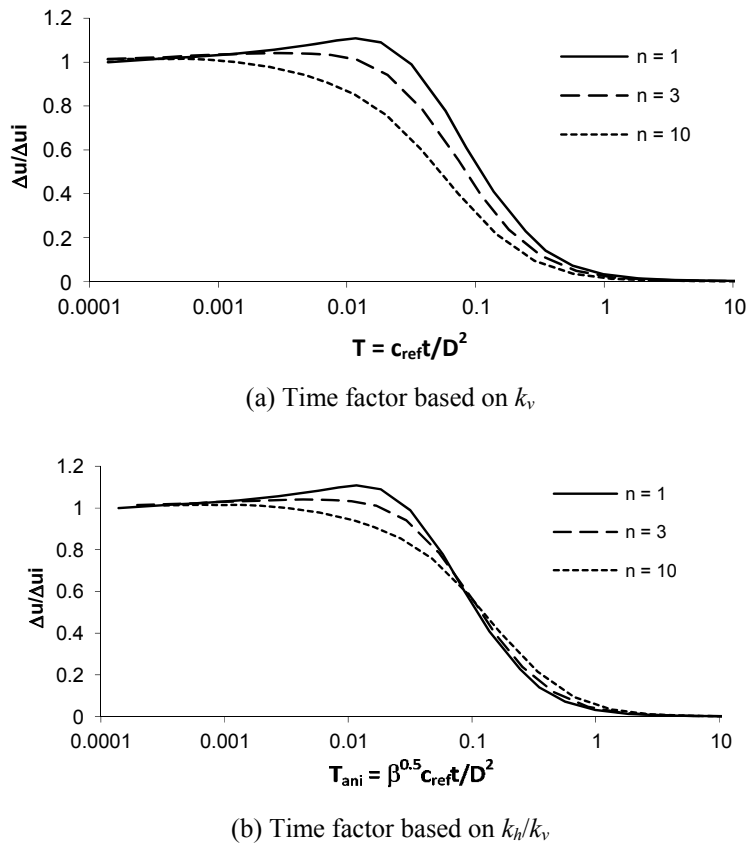


**Figure 6:** Normalised dissipation time histories from piezocone tests



**Figure 7:** Coefficient of consolidation as a function of vertical effective stress.

The preceding discussion assumed isotropic permeability. Many natural soils exhibit anisotropic permeability, typically with  $k_h > k_v$ . The effect of anisotropic permeability has been considered previously in solutions of elastic finite element analysis [17]. Figure 8 shows selected results from the study for ratios of horizontal to vertical permeability, i.e.  $n = k_h/k_v$  as defined in Section 4, of 1 (isotropic), 3 and 10. It can be seen that the Mandel-Cryer effect becomes less significant with increasing permeability anisotropy and as a result the gradient of the dissipation response becomes less steep over the central part.



**Figure 8:** Effect of permeability anisotropy on consolidation beneath a surface foundation

In the results shown in Figure 8, the permeability anisotropy ratio,  $n$ , was varied by increasing the magnitude of horizontal permeability while keeping the value of vertical permeability constant. The time for consolidation therefore reduces with increasing  $n$  due to the overall increase in the ‘resultant’ coefficient of (vertical and horizontal) consolidation. An adjustment can be applied to the time factor, or more strictly to the assumed coefficient of consolidation for isotropic conditions, to account for the relative increase in the resultant operative value of the coefficient of consolidation. An adjustment of the form suggested for the piezoball interpretation described in Section 4 is applied

$$T_{ani} = \frac{\beta^{0.5} c_{ref} t}{D^2} \quad (13)$$

where  $\beta$  is as given in Eq. 8 and  $c_{ref}$  is the operative coefficient of consolidation calculated as a function of the coefficient of vertical permeability. In the case of the shallow foundation consolidation response, an adjustment of  $\beta^{0.5}$  was found to provide the best fit with the numerically derived data. Use of Eq. 13 unifies the normalised time histories for the shallow foundation consolidation response, notwithstanding the initial variations in dissipation due to the Mandel-Cryer effect (Figure 8b).

## 6 DISCUSSION

The time-scale of consolidation processes within soil is determined from the (potentially anisotropic) permeability and the stiffness of the soil skeleton. The two facets result in a coefficient of consolidation, which will vary from one boundary value problem to another and the extent to which the stress-strain response of the soil during consolidation comprises plastic compression or quasi-elastic ‘reloading’ or swelling. In clays and other fine-grained soils, the permeability of the soil is a relatively strong function of the void ratio, but may be determined through conventional laboratory testing. However, estimation of an appropriate stiffness in order to derive a coefficient of consolidation is more challenging.

Three different boundary value problems have been considered here, all analysed using Modified Cam Clay to represent normally consolidated clay, with parameters suitable for the kaolin clay used to obtain corresponding experimental data. The three problems concern: (1) 1-D consolidation and swelling; (2) pore pressure dissipation around a piezoball penetrometer; and (3) pore pressure dissipation at the centre of a circular surface foundation. For the latter two problems, the effects of anisotropic permeability have also been considered.

Comparing Figures 1 and 6, deduced values of the coefficient of consolidation are bounded by the oedometric values for plastic compression and swelling, which differ by a factor of 6 or 7 for kaolin clay, compared with the  $\lambda/\kappa$  ratio of 4.7. One of the reasons for the greater factor obtained from numerical analysis is the effect of varying  $K_0$  (and hence ratio of  $p'$  to  $\sigma'_v$ ) during swelling as the soil becomes increasingly more over consolidated.

The consolidation coefficient relevant for pore pressure dissipation around a penetrometer was found to be a factor of about 5 greater than the oedometric compression value (for a given vertical effective stress). Although superficially comparable with the  $\lambda/\kappa$  ratio, parametric studies varying  $\lambda$  and  $\kappa$  separately suggest a more complex relationship (Eq. 6).

Data from model tests suggest an operative coefficient of consolidation for pore pressure dissipation from beneath a shallow foundation that lies approximately mid-way between values from laboratory oedometer tests or in situ piezocone (or piezoball) dissipation tests. This appears inconsistent with finite element solutions for soil modelled either as purely elastic or as normally consolidated Modified Cam Clay, where the operative consolidation coefficient from the latter approach appeared to coincide with the oedometric compression value.

Approaches to adjust the derived operational coefficient of consolidation to account for permeability anisotropy have also been suggested. For the piezoball dissipation, the coefficient of consolidation may be adjusted by simple averaging of the permeabilities in the vertical and horizontal directions. By contrast, for dissipation beneath a shallow foundation, the operative permeability appears to vary according to the square root of the average permeability. This seems logical, given that the latter problem is more dominated by vertical flow of pore water, hence the operative coefficient of consolidation is a weaker function of the ratio  $k_h/k_v$ .

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